Lidar multiple scattering models for use in cirrus clouds

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ABSTRACT

An approximate model for lidar multiple scattering has been extend to compute lidar returns when the cloud particle sizes are described by log-normal, gamma or exponential distribution functions. This paper presents examples computed for log-normal distributions. The work was motivated by ongoing efforts to remotely measure particle sizes in cirrus clouds which often contain a very wide range of particles sizes.

BACKGROUND

Multiple scattering causes the lidar return signal from clouds to increase with increasing receiver field of view. Calculations show that variation is strongly dependent on both the vertical variation of the scattering cross section and the width of the forward diffraction peak in the scattering phase function. The diffraction peak width is directly related to particle size. Thus, multiply scattered lidar returns can be used to determine the size of cloud particles. High Spectral Resolution (HSRL) and Raman lidars are particularly suited for this task because they provide direct measurements of the scattering cross section profile. The HSRL can also separately measure photons which have been backscattered by molecules while having undergone one or more forward scattering events on cloud particles. This isolates the effects of diffraction peak variations from variations in the near backscatter part of the phase function.

The recovery of particle size information requires a model of multiply scattered lidar returns. This can be accomplished with an equation based on the assumption that the diffraction peak can be represented as a Gaussian function of scattering angle (Eloranta, 1998). Cirrus clouds often contain particles with a very wide range of sizes. In this case, the Gaussian approximation may not provide a good approximation to the forward scattering peak. A new model which explicitly considers the width the particle size distribution is described.

A SUM OF GAUSSIAN DESCRIPTION OF THE DIFFRACTION PEAK

Following an approach similar to that used by Weinman(1976), the diffraction peak is represented with a sum of Gaussian. Assume that different portions of the scattering cross section are associated with different values of the Gaussian angular width, θ_s. In this case the forward diffraction peak can be expressed as a function of scattering angle, θ:

$$\frac{\mathcal{P}(\theta, z)}{4\pi} = \frac{1}{2\pi\beta_s(z)} \int_0^\infty \frac{1}{\theta_s^2(r,z)} \frac{d\beta_s(r,z)}{dr} exp\left(-\frac{\theta^2}{\theta_s^2(r,z)}\right) dr \tag{1}$$

where θ_s has been written as a function of particle size, r, and where the scattering cross section β is:

$$\beta_s(z) = \int_0^\infty \frac{d\beta_s(r,z)}{dr} dr \tag{2}$$

Where the mean-square scattering angle is defined in terms of particle radius and the lidar wavelength, λ as:

$$\theta_s^2(r,z) = \left(\frac{\lambda}{\pi r(z)}\right)^2 \tag{3}$$

.

When the scattering phase function is known, $\frac{d\beta(r,z)}{dr}$ can be selected such that eq. 1 provides the best fit to the phase function. However, when multiple scattering measurements are used to remotely sense particles sizes, the phase function is not known and it is desirable to express the diffraction peak in terms of particle size distribution parameters.

Log-normal distribution

The log-normal size distribution can be specified with just three parameters: 1) the total number of particles, 2) the effective radius, and 3) the width of the distribution. Consider a log-normal distribution, $n(r,z)$, of particle radius, r, which varies as a function of altitude, z, the particle size distribution can be written:

$$n(r,z) = \frac{a(z)}{\sqrt{2\pi}\gamma(z)r} exp\left(-\frac{1}{2}\left(\frac{ln\left(\frac{r}{\alpha(z)}\right)}{\gamma(z)}\right)^2\right); \tag{4}$$

When the particle size distribution, $n(r)$ is known and the efficiency factor, $Q(r,z)$,can be computed, the scattering cross section β can be computed :

$$\beta(z) = \int_0^\infty \pi r^2 Q(r,z) n(r,z) dr \tag{5}$$

Thus:

$$\frac{d\beta(r,z)}{dr} = \pi r^2 Q(r,z) n(r,z) \tag{6}$$

In cirrus clouds, particles are typically large compared to the lidar wavelength such that $Q(z) \approx 2$:

$$d\beta(r,z) = \frac{\sqrt{2\pi}a(z)r}{\gamma(z)} exp\left(-\frac{1}{2}\left(\frac{ln\left(\frac{r}{\alpha(z)}\right)}{\gamma(z)}\right)^2\right) dr; \tag{7}$$

The parameter α can be written in terms of the effective radius, $r_{eff} = \frac{<r^3>}{<r^2>}$ as:

$$\alpha(z) = r_{eff}(z) exp(-2.5\gamma(z)^2)$$

and the parameter a can be written in terms of β as:

$$a(z) = \frac{\beta(z)}{2\pi r_{eff}(z)^2} exp(3\gamma(z)^2) \tag{8}$$

With $\frac{d\beta}{dr}$ expressed in terms of $\beta(z)$, r_{eff}, and $\gamma(z)$ the integral $\int_0^\infty \frac{d\beta}{dr} dr$ can substituted for β each time it appears in the multiple scattering equation (eq 11 Eloranta, 1998). An additional integration must then be performed over particle size for each order of scattering. The resulting equation allows computation of the multiply scattered lidar return when vertical profiles of $\beta(z)$, $r_{eff}(z)$ and $\gamma(z)$ are provided.

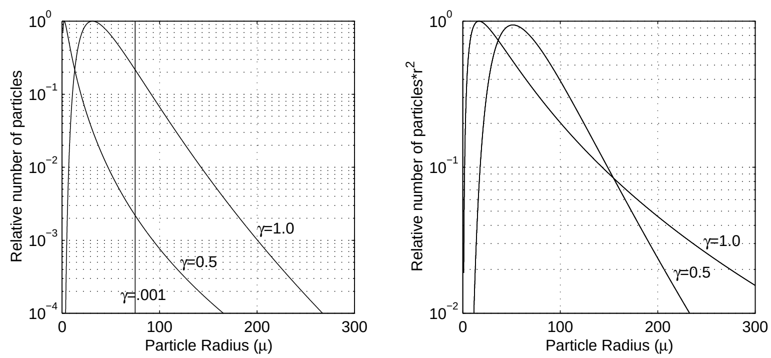

Figure 1. Log-normal size distributions of spheres with an effective radius of 75 microns and distribution width parameters(γ=0.001, 0.5, and 1.0). The relative number density is plotted in the left-hand panel and the particle area-weighted distribution on the right.

COMPARISONS WITH DIFFRACTION THEORY

Substituting equation 7 into equation 1 yields an expression for the diffraction peak phase function in terms of the effective radius and the width of the log normal size distribution. To illustrate the effect of the distribution width on the multiply scattered lidar return, we consider particle size distributions with three different widths and an effective radius of 75 microns. The left panel of figure 1 shows the relative number densities. The right panel shows the area-weighted size distributions in order to better illustrate the influence of particles on the scattering properties of the cloud.

Figure 2 compares the diffraction peaks in the scattering phase function derived from the sum of Gaussian approximation with those computed directly from the diffraction theory for circular apertures. Also shown are those computed with a single Gaussian using the effective radius in equation three to determine the angular width of the Gaussian. In this case and in the sum of Gaussian case the energy in the forward peak is constrained to be 1/2 of the total scattered energy. This forces the approximation to match the area required by diffraction theory. Increasing the width of the particle size distribution increases both small and large angle scattering at the expense of mid-range angle scattering. Comparisons of the phase functions(upper panel of fig 2) show very good fits between diffraction theory and the sum of Gaussian approximation. When the phase functions are weighted by θ^2 (to better represent the scattered energy) differences appear. As the width of the distribution increases, the diffraction rings which appear in the for the narrowest distribution average disappear. The sum of Gaussian model always matches the value for direct forward scatter. However, it over predicts at intermediate angles, and under predicts at large angles. This occurs because the Airy function describing diffraction provides more energy in at large angles than do the Gaussian making up the sum.

The influence of the log-normal distribution width on multiple scattering is shown in figure 3. As shown in figure 2 broadening the distribution increases the number of photons scattered at both small and large angles with a compensating reduction of the number of photons scattered at intermediate angles. The 2.2 mr FOV case shows marked reduction of signal with increased distribution width cause by loss of photons in the large angle tail of the angular distribution. In this case, changes in the small and intermediate angle portions of the angular distribution have no influence, because all of these photons remain within the receiver FOV. In contrast, the 0.11 mr case shows relatively little change with distribution width. This occurs because fewer photons are lost from the middle part of the angular distribution and the larger number of photons in the small angle part of the distribution compensate for the loses from the large angle part of the distribution.

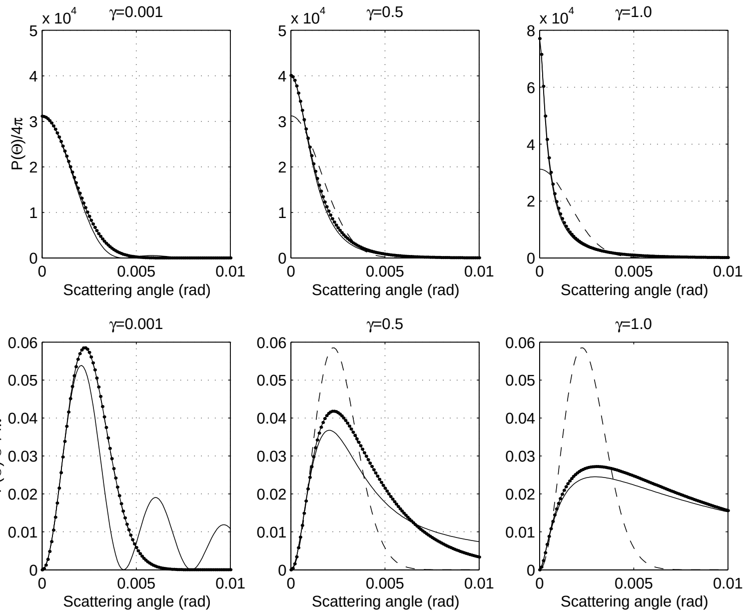

Figure 2. Phase functions for log-normal distributions of spheres with $r_{eff} = 75\mu$ (top). Diffraction peaks are plotted for log-normal width parameters of: γ=0.001, 0.5, and 1.0. Computations from diffraction theory are shown as a solid-line while the sum of Gaussian results are shown as a bold lines and the simple Gaussian result as a dashed lines. The bottom panel plots $P(\theta) \cdot \theta^2/4\pi$ to better reflect the scattered energy.

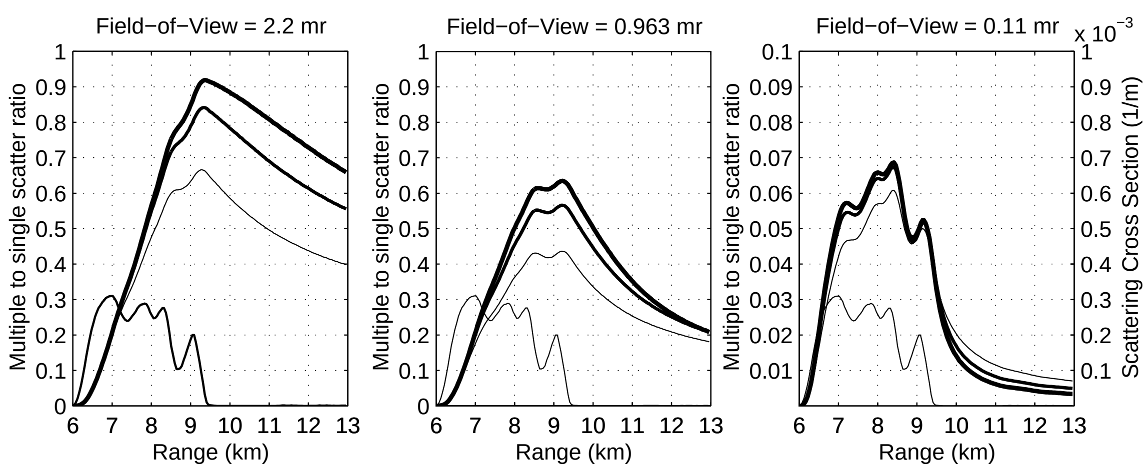

Figure 3. The ratio of multiply to singly scatter lidar returns as a function of altitude computed for log-normal distributions of particle size and receiver fields-of-view of: 2.2 mr (left panel), 0.963mr (center) and, 0.11mr (right). An r_{eff} of 75 microns was used for all computations. Results γ = 0.001 (bold), γ =0.5 (medium), γ =1.0 (light) are shown. The scale is expanded by a factor of 10 in the right panel. The scattering cross section profile is also shown as a light line in the lower left of all three panels with the scale shown on the right axis.

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